CORE MATHEMATICS (C) UNIT 1 TEST PAPER 3

1. Given that $(1 + 2^2 + 3^3 + 4^4)^{1/2} = c\sqrt{2}$, find the value of the integer c. [3]

2. A rectangular plot of land, of area 10 m^2, is to be enclosed by a fence of total length 16 m. If the plot has length x m and width y m, write down two equations in x and y and solve them to find the dimensions of the plot. Give your answers in surd form. [6]

3. Differentiate with respect to x:
   (i) $(2x - 5)^2$, (ii) $\dfrac{(2x-5)^2}{x^3}$. [7]

4. P is the point (1, 2) and Q is the point (3, 5).
   The point R lies on the line with equation $2y = x + 3$, and the angle PQR is a right angle.
   Find, as exact fractions, the coordinates of R. [8]

5. (i) Show that for all real values of k, the equation $x^2 + kx + (k - 2) = 0$ has real roots for x. [4]
   (ii) Find, in terms of k, the roots of the equation $x^2 + kx + (k - 1) = 0$. [4]

6. (i) Given that $16^x = 8^{2y-1}$, find the rational numbers a and b such that $y = ax + b$. [4]

(ii) Find the values of x and y which satisfy the simultaneous equations

$$16^x = 8^{2y-1}, \quad 3^{2x} = 9^{2-3y}.$$ [4]

7. The circle C has equation $x^2 + y^2 + 6x - 16 = 0$.

(i) Find the centre and the radius of C. [4]

(ii) Verify that the point A (0, 4) lies on C. [1]

(iii) Find the coordinates of D, given that AD is a diameter of C. [4]

8. A rectangular box is $(1-x)$ m wide, $(1+x)$ m long and $3x$ m high.

(i) Show that the volume of the box is $(3x - 3x^3)$ m^3. [2]

(ii) Find the value of x for which the volume is maximum. [4]

(iii) Justify that this value of x does maximize the volume. [2]

(iv) Express the maximum volume in the form $a\sqrt{b}$, where a and b are integers. [3]

9. In this question, $f(x) \equiv (x + 1)(x - 2)(x + 3)$.

(i) Sketch the graph of $y = f(x)$, showing the coordinates of all the points where the graph crosses the x-axis and the y-axis. [5]

(ii) Express $f(x)$ in its simplest form without brackets. [3]

(iii) Describe in words the transformations which would map the graph of $y = f(x)$ to that of

(a) $y = x(x - 3)(x + 2)$, [2]

(b) $y = x^3 + 2x^2 - 5x$. [2]

CORE MATHS 1 (C) TEST PAPER 3 : ANSWERS AND MARK SCHEME

1.	$(1+4+27+256)^{1/2} = \sqrt{288} = \sqrt{2\times 144} = 12\sqrt{2}$ $\quad c = 12$	M1 A1 A1	3
2.	$xy = 10,\ 2x + 2y = 16$	B1 B1	
	$x(8-x) = 10 \quad x^2 - 8x + 10 = 0 \quad (x-4)^2 - 6 = 0 \quad x = 4 \pm \sqrt{6}$	M1 A1 M1	
	Dimensions are $(4 + \sqrt{6})$ m by $(4 - \sqrt{6})$ m	A1	6
3.	(i) $d/dx\ (4x^2 - 20x + 25) = 8x - 20$	B1 M1 A1	
	(ii) $d/dx\ (4x^{-1} - 20x^{-2} + 25x^{-3}) = -4x^{-2} + 40x^{-3} - 75x^{-4}$	B1 M1 A1 A1	7
4.	Gradient $PQ = 3/2$, so gradient $QR = -2/3$	B1 B1	
	Equation of QR is $y - 5 = -2/3\ (x-3)$ $\quad 2x + 3y = 21$	M1 A1 A1	
	At R, also $2y - x = 3$, so $4y - 2x = 6$ $\quad y = 27/7$ $\quad R = (33/7, 27/7)$	M1 A1 A1	8
5.	(i) $b^2 - 4ac = k^2 - 4k + 8 = (k-2)^2 + 4$, which is > 0 for all real k	M1 A1 M1 A1	
	(ii) $x = \dfrac{-k \pm \sqrt{k^2 - 4(k-1)}}{2} = \dfrac{-k \pm (k-2)}{2}$ so roots are -1 and $1 - k$	M1 M1 A1 A1	8
6.	(i) $(2^4)^x = (2^3)^{2y-1}$ $\quad 4x = 6y - 3$ $\quad 6y = 4x + 3$ $\quad a = 2/3,\ b = 1/2$	M1 M1 A1 A1	
	(ii) $3^{2x} = 9^{2-3y}$ gives $2x = 4 - 6y$ $\quad x = 1/6,\ y = 11/18$	B1 M1 A1 A1	8

7.	(i) $(x+3)^2+y^2=25$ Centre $(-3, 0)$, radius 5	B1 M1 A1 A1	
	(ii) $0+16+0-16=0$	B1	
	(iii) $D=(-3-3, 0-4)=(-6, -4)$	M1 M1 A1 A1	9
8.	(i) Volume $=3x(1-x)(1+x)=3x(1-x^2)=3x-3x^3$	M1 A1	
	(ii) $\mathrm{d}V/\mathrm{d}x=3-9x^2=0$ when $x=1/\sqrt{3}$	M1 A1 M1 A1	
	(iii) $V''=-18x<0$, so max.	M1 A1	
	(iv) $V_{\max}=\sqrt{3}-\sqrt{3}/3=2\sqrt{3}/3$ m^3	M1 A1 A1	11
9.	(i) Curve crossing axes at $(-3, 0)$, $(-1, 0)$, $(2, 0)$, $(0, -6)$	B5	
	(ii) $\mathrm{f}(x)=(x+1)(x^2+x-6)=x^3+2x^2-5x-6$	M1 A1 A1	
	(iii) (a) $x\to x-1$, so translation 1 unit in positive x-direction	M1 A1	
	(b) $y\to y+6$, so translation 6 units in positive y-direction	M1 A1	12